The Valuation of Crack Spread Options with Jumps: Univariate Approach

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Outline

Overview

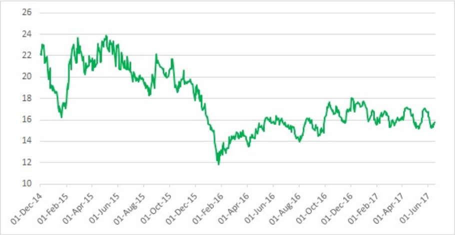
Crack spread

A difference between future prices of refined products and crude oil

- Determine the margin of refiners
- Used to manage the price risk spread
- Written as a crude-to-product ratio
- Traded at NYMEX

Crack spread options

Figure: Heating Oil/WTI Crack spread from December 2014 to June 2017



- Included in class of spread options
- Introduced at 1994
- Hedging margin
- Most liquid crack spread options: NY Harbour ULSD (heating oil) and NY Harbour RBOB (unleaded gasoline)

The specific characteristics of energy market

Figure: HO, WTI, and HO/WTI Crack spread from December 2014 to June 2017

- Possibility of jumps
- Volatile underlying assets and crack spread
- Cointegration relationships between crude oil prices and heating oil

The extension of existing valuation framework

- The application of option pricing on energy markets
- Addressing the specific spot model of commodity assets
- Comparing the univariate and explicit approach performances
- Most studies on crack spread options are European type
- Numerical approach on crack spread option pricing

Characteristics of commodity market

- Mean reverting (Bessembinder et al., 1995)
- Seasonality (Back et al., 2013; Paschke and Prokopczuk, 2007)
- Spot price model (Gibson and Schwartz, 1990; Schwartz and Smith, 2000)
- Cointegration (Duan and Pliska, 2004; Duan and Theriault, 2007; Dempster et al., 2008; Farkas et al., 2017)
- Jumps (Hilliard and Reis, 1999; Hilliard and Reis, 1998; Kyriakou et al., 2016)
- Stochastic volatility (Brooks and Prokopczuk, 2013; Chen and Ewald, 2017)

Valuation of spread option

- European spread option (Margrabe, 1978; Poitras, 1998; Carmona and Durrleman, 2003; Dempster and Hong, 2002; Duan and Pliska, 2004)
- European crack spread option (Mbanefo, 1997; Duan and Theriault, 2007; Dempster et al., 2008; Mahringer and Prokopczuk, 2015; Farkas et al., 2017; Aba Oud and Goard, 2016)
- American spread option (Jackson et al., 2007; Jaimungal and Surkov, 2008; Ziveyi, 2011; Chiarella and Ziveyi, 2013)
- American crack spread option (Mahringer and Prokopczuk, 2015)

Univariate and explicit models of spread options

- Univariate models (Dempster et al., 2008)
- Explicit models (Margrabe, 1978; Shimko, 1994; Carmona and Durrleman, 2003; Mbanefo, 1997; Alexander and Venkatramanan, 2007; Duan and Pliska, 2004; Duan and Theriault, 2007; Nakajima and Ohashi, 2012; Farkas et al., 2017)
- Comparing two models (Mahringer and Prokopczuk, 2015; Aba Oud and Goard, 2016)

Spread option and cointegration

- Spread option
  Let S_1 and S_2 are two asset price processes, K is the strike price, at maturity T, pay off function of European spread call option:

$$\max((S_1(T) - S_2(T) - K), 0)$$

- Cointegration
  Let $\mathbf{Y_1(t)}$ and $\mathbf{Y_2(t)}$ (t denotes time) are two variables, then the non stationary time series $\mathbf{Y_1(t)}_{t \in \mathcal{T}}$ and $\mathbf{Y_2(t)}_{t \in \mathcal{T}}$ are said to be cointegrated if the linear combination

$$\mathbf{Y_1(t)} - \alpha \mathbf{Y_1(t)}_{t \in \mathcal{T}}$$

  stationary for some $\alpha \in \mathbb{R}$
- The spread should be modelled directly if cointegration relation exists (Dempster et al., 2008))

The spot price model

- Let the spot spread process $\mathbf{x}$ and the long run factor $\mathbf{y}$ under the risk neutral measure satisfy:

$$\begin{aligned} \mathrm{d}\mathbf{x}_t &= [k(\theta + \varphi(t) + y_t - x_t)\mathrm{d}t + \sigma \mathrm{d}\mathbf{W} \\ \mathrm{d}\mathbf{y}_t &= (-k_2 y_t)\mathrm{d}t + \sigma_2 \mathrm{d}\mathbf{W}_2 \qquad . \\ E[\mathrm{d}\mathbf{W}\mathrm{d}\mathbf{W}_2] &= \rho \mathrm{d}t \end{aligned} \tag{1}$$

  where x and y are two latent factors with, long run means θ and 0. The k and k_2 are mean reversion speeds.

- The seasonality function $\varphi(t)$ induced by

$$\varphi(t) = \sum_{i=1}^{K} [\alpha_i \cos(2\pi i t) + \beta_i \sin(2\pi i t)] \tag{2}$$

  with α_i and β_i are constants. The dynamics of the spot spread $\mathbf{x}_t$ as reverting to a stochastic long run mean $\theta + \mathbf{y}_t$.

The option price (1)

- The two factor model futures spread:

$$
\begin{aligned}
F_s(t, T; x_t) = & x_t \mathrm{e}^{-k(T-t)} + \theta[1 - \mathrm{e}^{-k(T-t)}] \\
& \frac{y_t k}{k - k_2}[\mathrm{e}^{-k_2(T-t)} - \mathrm{e}^{k(T-t)}] + G(t, T).
\end{aligned} \tag{3}
$$

- The European call option price:

$$
C(F_s, t) = B \frac{b_s}{\sqrt{2\pi}} \exp\left[- \frac{(F_s - K)^2}{2b_s^2} \right] + B(F_s - K)\phi(\frac{F_s - K}{b_s}) \tag{4}
$$

where B is the price of discount bond, and F_s and b_s, respectively are the market observed futures spread and standard deviation of futures spread.

The option price (2)

The standard deviation b_s is given by:

$$b_s := \sqrt{A_1^F + A_2^F + 2\rho A_3^F} \tag{5}$$

where

$$
\begin{aligned}
A_1^F :=& \frac{\sigma^2}{2k}[\mathrm{e}^{-2k(T-R)} - \mathrm{e}^{-2k(T-t)}],\\
A_2^F :=& \frac{k^2\sigma_2^2}{(k-k_2)^2}\Big(\frac{1}{2k_2}[\mathrm{e}^{-2k_2(T-R)} - \mathrm{e}^{-2k_2(T-t)}] + \frac{1}{2k}[\mathrm{e}^{-2k(T-R)} - \mathrm{e}^{-2k(T-t)}]\\
&- \frac{2}{(k+k_2)}[\mathrm{e}^{-(k_2+k)(T-R)} - \mathrm{e}^{-(k_2+k)(T-t)}]\Big),\\
A_3^F :=& \frac{k\sigma\sigma_2}{k-k_2}\Big(\frac{1}{k+k_2}[\mathrm{e}^{-(k+k_2)(T-R)} - \mathrm{e}^{-(k+k_2)(T-t)}]\\
&- \frac{1}{2k}[\mathrm{e}^{-2k)(T-R)} - \mathrm{e}^{-2k(T-t)}]\Big)
\end{aligned}
$$

Univariate model with jumps (1)

- Jumps component is modelled in the futures prices dynamics
- The futures prices process

$$dF_J(t, t+\tau) = dF(t, t+\tau) + zdq_t - \lambda\mu_z dt \tag{6}$$

where q_t is a Poisson process with intensity λ, and the jump sizes, z are normally distributed with mean μ and standard deviation δ.

- The probability that futures price will be greater than exercise prices K,at the futures maturity T, conditional on $q_T = n$

$$\begin{aligned} & P\{F_J(T, T+\tau) > K | q_T = n\} \\ & \quad = P\Big\{a_s + \mu_z(n - \lambda T) + b_s\varepsilon_x + \delta\sum_{i=1}^{n}\varepsilon_i > K\Big\} \\ & \quad = P\left\{\frac{a_s - K + \mu_z(n - \lambda T)}{b_s^2 + n\delta^2} > -\varepsilon\right\} \\ & \quad = P\{d_2(n) > -\varepsilon\} \end{aligned}$$

Univariate model with jumps (2)

- ε is a standard normal random variable and $\varepsilon_x, \varepsilon_1, ..., \varepsilon_n$ are independent and standard normal random variables.

$$P\{d_2(n) > -\varepsilon\} = N(d_2(n)). \tag{7}$$

- Conditional on n jumps, the F_{JT} is normally distributed with mean and standard deviation

$$a_s(n) := a_s + \mu_z(n - \lambda T) \tag{8}$$

and standard deviation

$$b_s(n) := \sqrt{b_s^2 + n\delta^2} \tag{9}$$

- We have,

$$E[\varepsilon 1_{\{d_2(n)>-\varepsilon\}}]$$
$$= \int_{-d_2}^{\infty} \frac{\varepsilon}{\sqrt{2\pi}} e^{-\frac{1}{2}\varepsilon^2} d\varepsilon$$
$$= \frac{1}{\sqrt{2\pi}} \exp\{-\frac{1}{2}(d_2(n))^2\}.$$

- The price of call option price that expires at time R on a futures spread that expires at time T, c_J, at time t is given by

$$
\begin{aligned}
c_J &= \sum_{n=0}^{\infty} Prob(n) BE[(F_{JT} - K)^{+} | q_T = n] \\
&= \sum_{n=0}^{\infty} \frac{e^{-\lambda R}(-\lambda R)^n}{n!} BE[(F_{JT} - K)^{+} | q_T = n] \\
&= B \sum_{n=0}^{\infty} \frac{e^{-\lambda R}(-\lambda R)^n}{n!} E[(F_{JT} 1_{\{(F_{JT} > K\}} | q_T = n] - KE[1_{\{(F_{JT} > K\}} | q_T = n] \\
&= B \sum_{n=0}^{\infty} \frac{e^{-\lambda R}(-\lambda R)^n}{n!} (a_s(n)) N(d_2(n)) + b_s(n) \frac{1}{\sqrt{2\pi}} \exp\{-\frac{1}{2}(d_2(n))^2\} - KN(d_2(n)) \\
&= B \sum_{n=0}^{\infty} \frac{e^{-\lambda R}(-\lambda R)^n}{n!} (a_s(n) - K) N(d_2(n)) + b_s(n) \frac{1}{\sqrt{2\pi}} \exp\{-\frac{1}{2}(d_2(n))^2\}
\end{aligned}
$$

The popular existing univariate models (1)

- Bachelier model (ABM)

$$C(F_s,t) = e^{-r(T-t)}\left[(F_s - K)N(u) + \frac{\sigma\sqrt{T-t}e^{-\frac{u^2}{2}}}{\sqrt{2\pi}}\right], \qquad (10)$$

where $u = \frac{F_s - K}{\sigma\sqrt{T-t}}$ and $N(\cdot)$ is the cumulative standard normal distribution function. $\sigma^2 = \sqrt{(a\sigma_1)^2 - 2ab\rho\sigma_1\sigma_2 + (b\sigma_2)^2}$.

- Black-Scholes model (GBM)

$$C(F_s,t) = e^{-r(T-t)}\left[F_s N(d_1) - KN(d_2)\right] \qquad (11)$$

where $d_1 = \frac{\ln(\frac{F_s}{K}) + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$.

The popular existing univariate models (2)

- Schwartz one-factor model
  By assuming that $\sigma(t,T) = \sigma e^{-\eta(T-t)}$, the call option price formula is:

$$C(F_s, t) = e^{-r(T-t)}\left[F_s N(d_1) - K N(d_2)\right] \tag{12}$$

where

$$d_1 = \frac{\ln(\frac{F_s}{K}) + \frac{w^2}{2}(T-t)}{w\sqrt{T-t}},$$

$$d_2 = d_1 - w\sqrt{T-t} \quad \text{and}$$

$$w^2 = \frac{\sigma^2}{2\eta(T-t)}(1 - e^{-2\eta(T-t)}).$$

Data

- Cointegration and mean reversion test: futures prices
  - source: Reuters datastream
  - type: daily crude oil and Heating oil futures prices
  - period: January 2009 - May 2011
- Call option prices calibration: option prices
  - source: New York Merchantile Exchange (NYMEX)
  - type: daily Heating oil crack spread call option closing prices
  - period: February 2010 to April 2011

Statistics of futures prices data

The comparison of 1 month and 1 year futures spread

Figure: 1 month futures spread vs 1 year futures spread

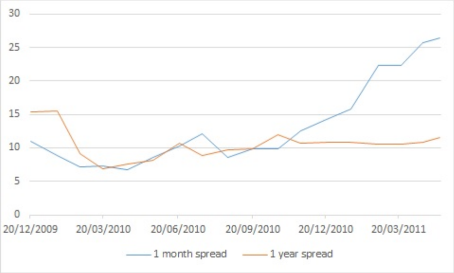


Figure: This figure shows the comparison of 1 month and 1 year futures spread within December 2009 to June 2011

Cointegration and mean reversion test

- Johansen cointegration test

Hypothesized No. of CE(s)	Eigenvalue	$p-value^{*}$	$p-value^{**}$
None*	0.3519	0.0324	0.0419
At most 1	0.0567	0.1586	0.1586

- Mean reversion test: estimating the regression model

$$\chi_L - \chi_S = \zeta + \gamma\chi_S + \varepsilon, \tag{13}$$

where χ_L and χ_S are, respectively, 1 year and 1 month spread level and ϵ is a noise term.

Table: Regression of mean reversion test

	ζ	γ
Values	9.61	-0.93
t-Stat.	7.64	-10.91
R-square	88.15%	

Option prices summary

- Two groups of day to maturity, $(R-t)$: M_1 is for $(R-t) \leq 60$ days and M_2 is for $(R-t) > 60$ days.
- Data summary

Table: Statistics of sample observation

	Call			Put		
	M_1	M_2	All	M_1	M_2	All
Number of observation	586	573	1159	573	870	1443
Average option price	1.82	1.44	1.63	0.58	1.14	0.92
Average spread prices	14.12	11.17	12.67	13.76	18.51	16.62

Minimizing the sum squared errors

- Let C_i and $\hat{C}_i$ are be the market and estimated prices of the crack spread call options contract i.
- e_i is the error of contract i, i.e.

$$e_i = \hat{C}_i - C_i.$$

- With $\theta(j)$ be the parameter vector for group M_j, $j = 1, 2$, for each pricing formula, the objective function is:

$$\min SSE(\theta(j)) = \sum_{i=1}^{N_j} e_i^2. \tag{14}$$

for prices data $(i = 1...N_j$, where N_j is the number of observations in group M_j. .

Model comparison

- The sum of squared errors
  For each group, M_1 and M_2, the sum of squared error (SSE) is used to compare errors in the performance of each model:

$$SSE = \sum_{i=1}^{N_j} (\hat{C}_i - C_i)^2, \quad j = 1, 2. \tag{15}$$

- The adjusted root mean squared errors
  For q is the number of pricing formula, the adjusted root mean squared errors is given by:

$$ARMSE = \sqrt{\frac{1}{N_j - q} \sum_{i=1}^{N_j} (\hat{C}_i - C_i)^2}, \quad j = 1, 2 \tag{16}$$

Estimated parameters, SSE and ARMSE for call option (1)

Estimated parameters	ABM	GBM	1-F Schwatrz	2-F Dempster	2-F Dempster with Jumps
$M_1(1 \leq (R-t) \leq 60, N_1 = 586$ observations)					
σ	10.2127	0.7320	0.8355		
η			2.6791		
k_1				1.7416	10.1379
k_2				6.8135	12.2175
σ_1				9.0101	9.4747
σ_2				8.3421	14.4978
ρ				-0.2804	-0.7985
μ					0.6324
δ					4.9156
λ					1.3366
SSE	16.053	17.7916	16.7174	15.0374	14.2145
ARMSE	0.1656	0.1743	0.1690	0.1603	0.1558

Estimated parameters, SSE and ARMSE for call option (2)

Estimated parameters	ABM	GBM	1-F Schwatrz	2-F Dempster	2-F Dempster with Jumps
$M_2((R-t)>60, N_2=573$ observation)					
σ	7.9996	0.6659	0.6659		
η			0		
k_1				3.9388	2.3375
k_2				0.5921	0.0155
σ_1				14.1400	7.0196
σ_2				20.9432	12.4432
ρ				-0.6564	-0.8221
μ					0.6162
δ					4.1271
λ					2.6819
SSE	17.6996	35.5006	35.5000	15.6750	12.7282
ARMSE	0.1758	0.2490	0.2484	0.1655	0.1491

Estimated parameters, SSE and ARMSE for put option (1)

Estimated parameters	ABM	GBM	1-F Schwatrz	2-F Dempster	2-F Dempster with Jumps
$M_1 (1 \leq (R-t) \leq 60, N_1 = 573$ observations)					
σ	9.5771	0.6860	0.8956		
η			5.4587		
k_1				0.6214	3.7459
k_2				3.2013	1.5114
σ_1				10.6954	9.5758
σ_2				9.2141	2.2650
ρ				-0.4936	-0.3542
μ					-0.0922
δ					6.1673
λ					0.8290
SSE	17.7718	26.5872	23.7624	16.7249	16.4322
ARMSE	0.1500	0.1835	0.1734	0.1455	0.1442

Estimated parameters, SSE and ARMSE for put option (2)

Estimated parameters	ABM	GBM	1-F Schwatrz	2-F Dempster	2-F Dempster with Jumps
$M_2((R-t)>60, N_2=870$ observation)					
σ	8.4221	0.5084	0.6906		
η			1.2866		
k_1				5.5147	2.2671
k_2				0.8135	0.7493
σ_1				16.4092	3.3306
σ_2				16.1458	8.5076
ρ				-0.5428	-0.2495
μ					-0.038
δ					8.3559
λ					1.2479
SSE	89.1103	73.1199	49.0401	76.2728	44.5769
ARMSE	0.3206	0.2904	0.2378	0.2966	0.2267

Market - models prices comparison and moneyness

Market and models prices comparison

- The average signed percentage error, SPE
  For each option prices, the SPE is given by

$$SPE = \frac{\hat{C}_i - C_i}{C_i} \times 100\% \tag{17}$$

- The average unsigned percentage error, UPE
  For each option prices, the UPE is given by

$$UPE = \left| \frac{\hat{C}_i - C_i}{C_i} \right| \times 100\% \tag{18}$$

Moneyness, $M = ln\frac{F_s}{K}$, group:

- in the money: $M > 0.05$
- at the money: $-0.05 \leq M \leq 0.05$
- out the money: $M < -0.05$

Market-model comparisons based on SPE and UPE for call option

Model	Av. SPE			Av. UPE		
	OTM	ATM	ITM	OTM	ATM	ITM
$M\_1(1 \leq (R-t) \leq 60)$						
ABM	-0.78%	-0.94%	0.27%	29.58%	16.72%	4.76%
GBM	-23.92%	-3.26%	-2.56%	29.26%	16.64%	5.41%
1-F Schwatrz	-17.71%	0.68%	-2.04%	26.31%	14.81%	4.96%
2-F Dempster	4.46%	2.08%	0.71%	28.67%	16.39%	4.53%
2-F Dempster with Jumps	5.39%	-0.28%	1.25%	26.87%	16.83%	4.31%
$M\_2((R-t) > 60)$						
ABM	-2.98%	1.10%	-2.40%	15.94%	7.84%	7.18%
GBM	-10.30%	-0.14%	7.05%	25.96%	7.38%	7.37%
1-F Schwatrz	-10.30%	-0.14%	7.05%	25.96%	7.38%	7.38%
2-F Dempster	-2.86%	0.78%	-2.47%	13.97%	7.12%	7.48%
2-F Dempster with Jumps	5.41%	-0.83%	-0.12%	13.10%	6.39%	5.74%

Market-model comparisons based on SPE and UPE for put option

Model	Av. SPE			Av. UPE		
	OTM	ATM	ITM	OTM	ATM	ITM
$M_1(1 \leq (R-t) \leq 60)$						
ABM	2.81%	-6.58%	-5.88%	39.80%	20.72%	8.14%
GBM	-24.64%	-11.41%	-8.88%	57.92%	25.55%	8.88%
1-F Schwatrz	-21.00%	-4.95%	-4.97%	56.26%	17.93%	6.34%
2-F Dempster	6.53%	2.22%	-2.97%	40.64%	20.56%	7.93%
2-F Dempster with Jumps	11.99%	-0.57%	-4.00%	39.26%	21.34%	8.30%
$M_2((R-t) > 60)$						
ABM	-24.66%	11.86%	10.55%	45.07%	17.67%	11.06%
GBM	-9.53%	-9.74%	-9.42%	28.35%	17.37%	11.70%
1-F Schwatrz	-0.45%	1.91%	-2.75%	21.68%	15.62%	6.78%
2-F Dempster	-24.43%	5.57%	4.87%	41.11%	12.35%	7.19%
2-F Dempster with Jumps	5.28%	-4.49%	4.60%	18.38%	11.50%	7.48%

Conclusion

- We proposed univariate crack spread option model with jumps that address cointegration
- The empirical analysis on futures prices shows the existence of cointegration relationship between futures prices of crude oil and heating oil.
- The two-factor Dempster with jumps model outperforms other models in short and long term time to maturity.
- Our proposed model also provide a lowest unsigned percentage error compare to other model for longer time to expiry.

The further research covers:

- The cracks spread option pricing with explicit approach,
- The american crack spread option pricing

Terima kasih

Thank you

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