RELIABILITY BASED DESIGN APPROACH TO STOCHASTIC SUPPLY PLANNING

Matthias Ondra

Vienna University of Technology, Institue of Management Science, Theresianumgasse 27, 1040 Vienna, Austria

Matthias.ondra@tuwien.ac.at

Abstract

Due to structural changes in the electricity market like the ongoing integration of renewable energy technologies, various aspects in energy planning problems become increasingly volatile. In recent years new methods concerning the integration of stochastic components in energy planning models have become popular. Whenever renewable energy technologies are integrated in energy models, energy managers have to be concerned if these energy assets can be considered as reliable power sources. This paper explicitly considers these issues related to system reliability and proposes a stochastic optimization framework to integrate the estimation of the reliable demand that can be provided by implementing the methodology of reliability based design optimization. The basic building block of this stochastic framework, covering the integration of renewable energy technologies is discussed in detail and a short outlook of possible extensions is given. Within the scope of the model, different approaches of modeling the total energy that can be supplied from an energy park based on probabilistic descriptions (Weibull, Exponential, Beta and Log-normal) are compared to the assumption of Gaussian distributions, which is considered as the benchmark model. This paper gives evidence, that modeling the power available via adapted probability density functions can outperform the benchmark model in the regime of higher levels of reliability.

1 Introduction

The integration of renewable energy sources (RES) in energy planning problems has led to an increased focus to incorporate risk management approaches in energy strategies. Besides the compelling advantages of RES technologies resulting in decreased pollution and simultaneously presenting economic feasible solutions a major problem remaining is, if RES can be considered as reliable power sources. Uncertainty in the output is often addressed to be the main problem associated with RES (Hemmati et al., 2017). The task of supplying a predefined load with high reliability including RES in an economic manner seems challenging and is in fact a key factor in energy planning problems (Monishaa et al., 2013). Energy planners need to be aware of the energy assets' capability to provide a certain supply. The difficulty in accordance with this problem is that the power available from RES fluctuates due to variations in the weather conditions (Delucchi and Jacobson, 2011). With a focus towards the two most used self-generating technologies nowadays, wind speed affects the power output of wind turbines and solar irradiance affects the power output of photovoltaic systems, (Nojavan et al., 2019). In energy planning problems, an approach to deal with these issues is given by imposing the relevant variables included in the model to be volatile. This stochasticity can be imposed on both supply and demand side and consequently results in a paradigm shift from deterministic to stochastic supply-demand constraints. This point of view has the benefit of explicitly addressing problems associated with risk management in energy strategies but opens up new problems ranging from computational complexity of the optimization problem to specific modeling aspects.

Recently, various authors (Yu et al., 2009; Monishaa et al., 2013; Beraldi et al., 2017b,a; Hemmati et al., 2017; Garifi et al., 2018; Huang et al., 2018; Huo et al., 2019; Ondra and Hilscher, 2019) addressed the stochasticity in energy planning models by imposing the reliability based chance constraint paradigm, which was introduced in (Charnes and Cooper, 1959). From this point of view, the stochastic supply-demand constraint has to be valid with a certain probability which is expressed in terms of the level of reliability. This ex-ante chosen level of reliability reflects the energy manager's attitude towards risk, which can be directly incorporated in the model formulation. The reliability based design optimization (RBDO) methodology is valuable because it has a dual goal, namely guaranteeing performance as well as system reliability under uncertainty (Geletu et al., 2013).

A stochastic formulation of the supply-demand constraint including a reliability level has been used recently in several papers covering a wide range of problems. Beraldi et al. (2017b) considered the procurement problem under reliability constraints with uncertainty on the demand-side. This contribution was extended by the authors (Beraldi et al., 2017a) who included also volatile market aspects via stochastic purchasing and selling prices. Monishaa et al. (2013) investigated cost effects of the generation expansion problem in a probabilistic constraint regime, where the power system loads were assumed to be Gaussian. Yu et al. (2009) considered a transmission network expansion planning problem including uncertainties on the supply side of wind turbine generators, where the wind speed was modeled via a Weibull distribution and a linearization of the power curve was used. Huo et al. (2019) included reliability chance constraints in an energy hub optimization problem, where the authors considered RES and modeled solar energy via Beta distribution and wind power via Weibull distribution. The solution of the chance constrained problem was approximated using the Cornish-Fisher expansion to estimate the resulting quantiles based on the quantiles of a normal distribution. Garifi et al. (2018) applied a chance constrained formulation to the demand response problem in a home energy management system, with uncertainty on the supply side. The authors modeled photovoltaic and wind power available by weather forecasts, where the prediction errors were assumed to be normally distributed. Hemmati et al. (2017) incorporated stochastic planning to energy storage systems, where solar and wind power was modeled based on Gaussian distributions. Huang et al. (2018) used a reliability chance constrained formulation of a demand response application, where the demand response errors were assumed to be normally distributed. Ondra and Hilscher (2019) extended the integrated portfolio investment model of Delarue et al. (2011) in the RBDO context to quantify risk diversification effects in renewable energy technologies.

Generally, a crucial point in the design of stochastic energy models is the choice of the underlying probabilistic structure of the stochastic variables. Various papers assume that the random variables are Gaussian. Using stochastic models which are not adapted to the problem, or ignoring the stochastic character involved in the model may lead to procurement plans which are infeasible on the one hand or overly expensive on the other hand (Beraldi et al., 2017a). A question remaining is if Gaussian probabilities can be considered as an approximation or are too simplistic in applications. Other choices than Gaussian probability density functions however, can result in immoderate technical problems in the optimization models. Flexible approaches which deal with reliability constraints are needed to account for a general framework to integrate RES in energy planning problems.

The aim of the paper is twofold. Firstly, this paper proposes a general framework to incorporate reliability constraints within the formulation of energy planning problems including RES, by using the RBDO approach. Frequently, issues related to system reliability involves the estimation of quantiles associated with the probability of the power from energy assets. A particular focus is put on the the model's fundamental building block which deals with this estimation by proposing a proactive stochastic planning model to evaluate the maximum reliable supply which can be provided by RES within the planning period of one year. The risk parameter accounting for reliability itself is specified within the methodology of RBDO. The flexibility of this approach is illustrated by means of two examples. This underlines the modular character of the framework and shows that this the framework can be easily adapted to various problems in stochastic energy planning. Secondly, we give evidence that adapted probabilistic formulations can outperform the case of Gaussian energy assets. Therefore, the model is demonstrated in a use case with photovoltaic power sources as well as wind turbine generators. Different models (Weibull, Exponential, Beta and Log-normal) for wind and solar power available are provided and validated in a backtest simulation compared to the benchmark model of Gaussian energy assets, which shows that the framework introduced is valuable.

The rest of the paper is organized as follows. Section 2 introduces the construction, illustrates further examples and deals with the calibration of the model parameters. The stochastic optimization problem is solved analytically for the benchmark model and is numerically approximated for the other underlying probability distributions. Section 3 reports on the computational experiments and validates the models in a backtest simulation. Section 4 gives a final conclusion of the results.

2 The energy model in the context of reliability engineering

We formulate the stochastic energy model to estimate the total reliable supply which can be provided by the RES of a wind generator and photovoltaic panels. The model is demonstrated in the use case, where the installed capacities of the energy assets are $\kappa_1 = 1\,\text{MW}$ for wind power and $\kappa_2 = 1\,\text{MWp}$ for solar power, respectively. The notion of a reliable demand is specified in the RBDO approach and quantified by the level of reliability $\chi \in [0.5, 1)$. In the RBDO setting, the constraint specifying the supply-demand relation is given by the supply shortage function i.e. the residual of the secured supply s_t and the power P_{it} available from the i-th energy asset at decision time $t \in T$. Here T denotes the equidistant set of decision points, divided into elementary weekly periods.

The regime of a negative supply-demand imbalance denotes the case, where more power is available to satisfy the demand, whereas the regime of a positive power gap denotes the case of less power than needed being available. Within the uncertain future of the amount of power that can be supplied, this constraint is considered as a stochastic supply-demand constraint. In the energy model, the power available from a specific energy asset is modeled as a random variable with an underlying probability distribution. The probability constraint can be referred to the "continuous probability density version" to compute the loss of load probability (LOLP), which can be used as a statistical metric to measure critical shortages in energy planning problems.

2.1 Model construction

The main focus in this paper is put on the construction of the framework's basic building block which integrates renewable energy technologies and addresses problems related to their ability to provide a reliable supply. With this framework's fundamental module, various problems can be easily set up and adapted. Since all these extension essentially rely on the on the properties related to the basic model, the latter is studied in detail. The basic energy manager's problem considered is to maximize the performance of the renewable energy technologies, by maximizing the reliable power that can be supplied subject to the stochastic chance constraint which has to hold true with an ex-ante chosen level of reliability χ. This model specific parameter acts as a threshold on the supply shortage function and constitutes the LOLP. Simultaneously, the reliability parameter reflects upon the energy planner's attitude towards risk $\varepsilon = 1 - \chi$. From a statistical point of of view, the probability constraint imposes a limit on the power that can by supplied, by requiring that s_t is the ε-quantile of the distribution of the total power available from both energy assets. Naturally, higher levels of system reliability or lower levels of risk respectively, decreases the secured supply. The secured supply is assumed to be constant on the supported intervals of the time grid. This mathematical formulation of the chance constrained problem (CCP), given by Eq.(1).

$$\text{CCP:} \qquad \max_{s_t \in \mathbb{R}_+} s_t, \quad \text{s.t.,} \quad \Pr\{s_t - P_{1t} - P_{2t} \leqslant 0\} \geqslant \chi, \quad \forall t \in T. \tag{1}$$

As there are also other ways to estimate the quantile of the distribution, we emphasize the flexibility of this approach since the model can be expanded in many ways to account for a great variety of problems in stochastic procurement planning, where other design frameworks reach their limit. This can be the case, whenever decision variables are included in the supply-demand constraint and thereby change the probability density function of the total power available. We give an outline, how the CCP can be easily adapted to other problems of interest. For the sake of an example consider the problem of optimally allocating a dispatchable supply security $P_{3t} = c$ over the planning period, where λ_t denotes the fraction of the power used at point of time t. The mathematical formulation of this chance constrained problem can easily adapted from (1) and is given by

$$\begin{aligned} \text{CCP1:} \qquad \max_{s_t, \lambda_t \in \mathbb{R}_+} \sum_{t \in T} s_t, \quad \text{s.t.,} \quad & \Pr\{s_t - P_{1t} - P_{2t} - \lambda_t c \leqslant 0\} \geqslant \chi, \\ & \sum_{t \in T} \lambda_t \leqslant 1, \qquad s_- \leqslant s_t \leqslant s_+, \qquad \forall t \in T. \end{aligned} \tag{2}$$

Another example covering the generation expansion problem, is to determine the minimal costs ξp needed to satisfy a prespecified demand d with a certain reliability, where ξ represents the change in capacity of solar panels and p denotes the associated costs. Again the mathematical formulation is given by adopting the fundamental CCP to

$$\text{CCP2:} \qquad \min_{\xi \in \mathbb{R}_+} \xi p, \quad \text{s.t.,} \quad \Pr\{d - P_{1t} - \xi P_{2t} \leqslant 0\} \geqslant \chi, \qquad \xi \geqslant 1, \qquad \forall t \in T. \tag{3}$$

These extensions (2) and (3), which are briefly discussed for the sake of completeness in the appendix, essentially rely on the properties related to the basic model (1). Therefore this paper focuses on the study of the basic model (1).

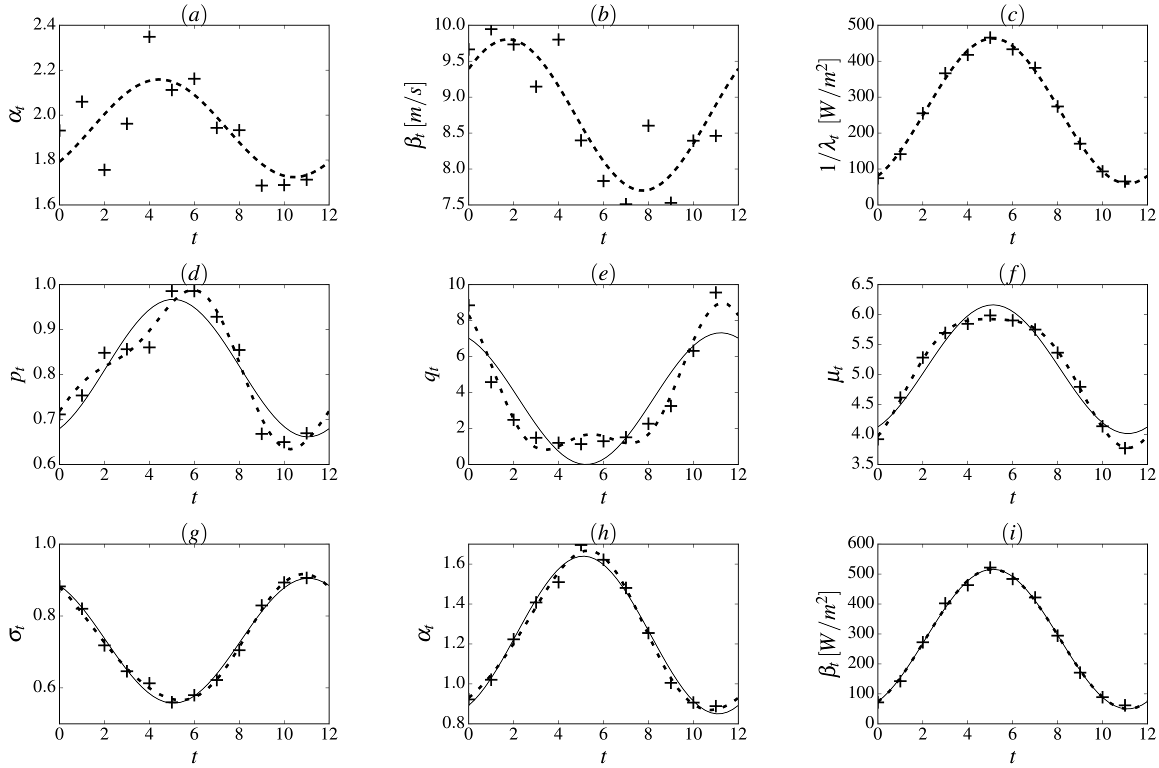

Figure 1: *Plot (a) shows the shape parameter α_t and (b) the scale parameter β_t of the Weibull distribution to fit the windspeed over the time period of one year. Concerning wind speed, the R-package Riem has been used, which provides windspeed from the ASOS wind station Schwechat, AUT in the time span from 01.01.2012-31.12.2016 in the daytime. Plots (c)-(i) show the parameters of different distributions used to fit the solar irradiance. In (c), the inverse rate parameter of the exponential distribution is given, (d) and (e) represent the parameters of a Beta distribution, (f) and (g) are the parameters of a log-normal distribution and (h) and (i) are the shape and scale parameter of the Weibull distribution. The data are from CAMS and collected vie the R-package CamsRad in the same time span.*

2.2 Model calibration

Wind & Solar Power. Wind speed is frequently modeled using a Weibull distribution with shape parameter α_t and scale parameter β_t. The wind speed following this distributions is then mapped to the power of wind turbine generators via the linearized power curve of the form

$$P_1(v) = \begin{cases} 0, & \text{for} \quad v \leqslant v_{\text{CI}} \quad \text{and} \quad v > v_{\text{CO}} \\ \frac{\kappa_1}{v_{\text{RO}} - v_{\text{CI}}}(v - v_{\text{CI}}), & \text{for} \quad v_{\text{CI}} \leqslant v \leqslant v_{\text{RO}} \\ \kappa_1, & \text{for} \quad v_{\text{RO}} \leqslant v \leqslant v_{\text{CO}}, \end{cases} \tag{4}$$

where $v_{\text{CI}} = 3m/s, v_{\text{RO}} = 11m/s, v_{\text{CO}} = 25m/s$ denote the cut-in-speed, rated-output-speed, cut-out-speed and κ_1 denotes the installed respectively. The power available from solar panels is given by $P_2 = I/I_{1000}\kappa_2$, where I denotes the solar irradiance, $I_{1000} = 1kW/m^2$ denotes the reference irradiance and κ_2 is the installed capacity of

Table 1: *Estimation of the parameters of the different models used in the simulation experiments, for both first and second order Fourier Series approximation.*

	Solar Power/Irradiance									Wind Power/Speed			
	(M0)		(M1)		(M2)	(M3)		(M4)		(M0)		(M1)-(M4)	
Var.	μ_t^{P}	σ_t^{P}	α_t	β_t	$1/\lambda_t$	p_t	q_t	μ_t	σ_t	μ_t^{P}	σ_t^{P}	α_t	β_t
a_0	261	196	1.24	283	261	-0.81	3.66	5.1	0.73	522	196	1.94	8.75
a_1	-181	37.5	-0.35	-208	-181	-0.13	3.36	-0.97	0.15	37.5	37.5	-0.15	0.64
a_2	-3.7	-17.9	0	-0.55	-4	0.04	1.28	-0.15	0	-2.4	-17.9	-0.02	-0.05
b_1	90	47.1	0.18	104	90	0.07	-1.45	0.47	-0.08	63	47.1	0.15	0.83
b_2	1.4	12.8	0	-0.62	1	0.03	-1.07	0.18	-0.02	2.1	12.8	-0.02	0.11
RMSE													
$(N=1)$	7.67	15.75	0.04	9.14	7.67	0.04	1.30	0.17	0.01	35.99	10	0.13	0.42
$(N=2)$	7.13	2.41	0.02	9.12	7.13	0.02	0.54	0.04	0.01	35.92	6.8	0.11	0.41

the solar panels. When it comes to modeling the stochastic character of solar irradiance, various models besides a normal distribution are used in the literature. The most recognized being: Weibull, Exponential, Beta and Log-normal distributions. In this paper different approaches to model the solar power available in the energy model are considered. The models using a Weibull, Exponential, Beta and Log-Normal distributions to model the solar irradiance are denoted by M1, M2, M3 and M4, respectively. In each of the models M1-M4, except for the benchmark model, wind speed is assumed to be Weibull distributed. The benchmark model M0 itself is given by Gaussian distributions of both power assets available. Historical weather data are fitted according to the underlying distribution via the moment matching approach to estimate the different parameters.

Dynamic effects in the parameters. To account for seasonal effects in the stochastic energy model, the dynamic evolution in the parameters is interpolated by a Fourier series of the form

$$\theta_t = a_0 + \sum_{n=1}^{N'} a_n \cos\left(\frac{n\pi}{52}t\right) + b_n \sin\left(\frac{n\pi}{52}t\right), \tag{5}$$

where N' is the order of the Fourier approximation, θ_t represents a parameter of the distributions and t is the point of time, given in weeks. In the analysis, we approximate the seasonal character of the parameters by first and second order Fourier approximations. The root mean squared error (RMSE) is used to evaluate the goodness of fit of the Fourier approximation. The results of the parameter fits are given in Tab.1.

2.3 Solution of the energy model

The benchmark model is constituted by the assumption of individual Gaussian distributions $P_{it} \sim N(\mu_{it}, \sigma_{it})$ of both wind $(i = 1)$ and solar $(i = 2)$ power available. The probability constraint of the stochastic optimization problem

(1) can be written as an analytic inequality in terms of the χ-quantile z_χ of the standard normal distribution, which has to hold true in every point of time during the planning horizon

$$s_t \leqslant \mu_t - \sigma_t z_\chi. \tag{6}$$

Here, μ_t and σ_t denote the mean and the standard deviation of the total power available (i.e. the sum of wind and solar energy assets), which are given by the aggregation law of normal distributions. In the light of the maximization problem, the inequality is sharp and is given by the expected power available subtracted by the term $\sigma_t z_\chi$ which accounts for risk correction, depending on the energy managers risk aversion. Consequently, this risk correction which decreases the secured supply is high, whenever either the standard deviation of the total power available or the level of reliability is high. It is due to the fact, that the feasible region is restricted to positive values to account for a valuable proactive planning strategy, that the maximal secured supply is given by

$$s_t = \max\{0, \mu_t - \sigma_t z_\chi\}. \tag{7}$$

The restriction on the positivity of the solution imposes an upper bound on the model parameter of the achievable level of reliability χ^* and can be rewritten in terms of the coefficient of variation $\chi^* \leqslant \Phi(1/c_v)$, where Φ denotes the cumulative distribution function of the standard normal distribution. High values of c_v denotes the regime of more unreliable planning strategies due to the high variance. Low values of c_v result in situations where the maximum level of reliability is increasing tremendously.

In case the distributions of the the power available are not given by a Gaussians, the chance constraint in Eq.(1) cannot generally be written in closed form. The solution is computed numerically, by applying probabilistic relaxation techniques to the original problem which are in line with robustness against the vast majority of possible scenarios, see (Calafiore and Campi, 2005). In this sample approach, the original chance constrained problem (1) is transferred to the associated sample convex program SCP_N, where the solution is randomized via sampling N constraints

$$\begin{aligned} \mathrm{SCP}_N: \quad & \max_{s_t \in \mathbb{R}_+} s_t \quad \text{s.t,} \\ & s_t - P_{1t}^{(k)} - P_{2t}^{(k)} \leqslant 0, \qquad \forall t \in T, \quad k = 1, \dots, N. \end{aligned} \tag{8}$$

In this expression, $\{P_{1t}^{(1)}, \dots, P_{1t}^{(N)}\}$ denotes the samples drawn from the wind distribution and $\{P_{2t}^{(1)}, \dots, P_{2t}^{(N)}\}$ denotes the samples drawn from the solar distribution, respectively. Calafiore and Campi (2005) provided a feasibility result that the scenario solution of the associated sampled program (8) is then feasible for the vast majority of unseen constraints, given that the sample size is specifically large, depending on the level of reliability.

Calafiore (2010) considered this fact as a generalization property in the learning-theoretic sense based on a training set of sampled constraints. In the subsequent work of Calafiore and Campi (2006) this sample size has been refined, as well as new sample and discard algorithms have been introduced in Campi and Garatti (2011), which are applied to solve to construct the associated SCP_N (8). This approach is highly suitable to applications considering general probability distributions, since it holds true irrespective of the distribution.

3 Discussion of the computational experiments

This section reports on the results of the different modeling approaches to predict the maximum reliable supply that can be provided by RES technologies and studies how dynamic peculiarities of specific models aggregate to a cumulative effect over the planning horizon. In case of the benchmark model M0 using Gaussian energy assets, the analytic solution for every point in time is given by (7) and in the other cases concerning the models M1-M4 assuming different probability distributions for the solar power, the solution is approximated via the sample approach (8). These stochastic models are compared with the optimal ex-post solution, which is calibrated using the weather data of 2017. A retrospective validation based on a backtest simulation, where the different models are reviewed on their capability to reproduce the optimal ex-post solution, is conducted. To measure the performance in the validation process, the coefficient of variation of the root mean square error CV(RMSE) (for a discussion of different validation measures in energy models, see (Aman et al., 2014)) is used

$$CV(RMSE) = \frac{1}{\bar{o}}\sqrt{\frac{\sum_{i=1}^{n}(p_i - o_i)^2}{n}}, \tag{9}$$

where $\bar{o}$ is the mean of the n observed values, p_i is the model predicted value. To study dynamic effects according to the different models, the results of the simulations carried out for each point in the planning period for the second order Fourier approximations are given in Fig.2(a)-(e) for all levels of reliability considered, $\chi \in \{0.5, 0.6, 0.7, 0.8, 0.9, 0.92, 0.94\}$. The plot (a) shows the maximal secured supply at each point in time subject to the specified level of reliability for the benchmark model of both simulated and analytic solution. This also illustrates the applicability of the simulation approach, as can be seen by comparing the analytic solution with the simulated version using the sample approach in (8). The simulation provides a good approximation of the analytic solution, with an average normalized CV(RMSE) of approximately 4% over all reliabiliy levels considered. The solution for the reliability level of $\chi = 0.5$ corresponds to the mean total energy of the optimal energy strategy, since the term accounting for risk correction equates to zero and does not affect the solution, see Eq.(7).

All of the models considered in Fig.2(a)-(e) show a distinct decrease of the supplied power with increasing levels of reliability. This behavior is reflected in every model, the magnitude of the effect however, varies according to

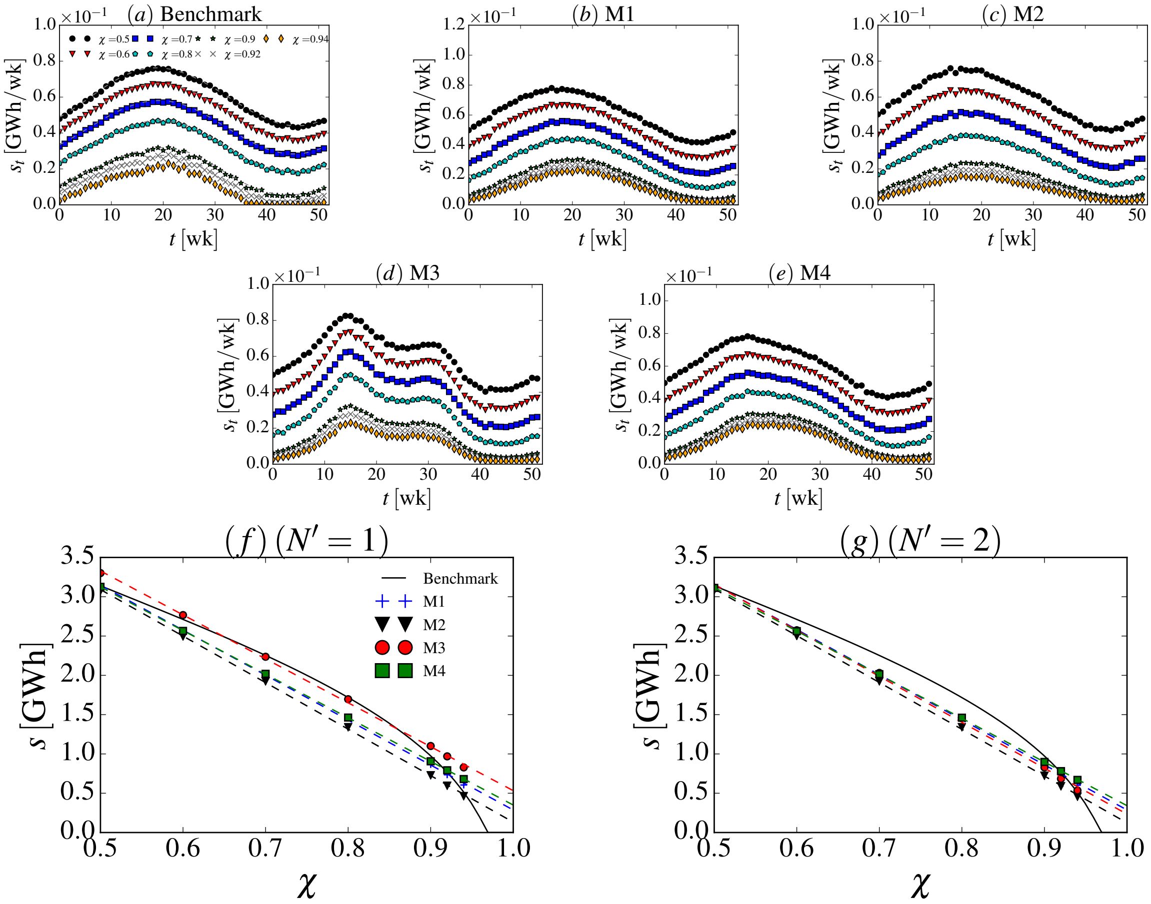

Figure 2: *(a)-(e) show the total secured supply over one year considering a ex-ante level of reliability, for the stochastic models. (g) and (h) show the cumulative effect of the total secured supply in one year by the different models for both first and second order parameter fit.*

the different models.

Model M3 (Fig.2(d)) assuming a Beta distribution of the solar power, shows a slightly different dynamic structure compared to the other models. It is a peculiarity of this model to predict higher amounts of solar energy in the summer months compared to the other models. This results in a higher predicted secured supply by solar energy assets in the summer time, as it can be seen in the plot by an increased crest of the wave.

To evaluate how these dynamic peculiarities aggregate to a cummulative effect in the planning period, we consider the total energy that can be supplied by an aggregation in the time dimension over the planning period. This total energy supply within the planning period of one year, denoted by s, carries over the structural dependence on the reliability parameter and hence also varies according to the energy manager's attitude towards risk.

The nonlinear influence of the energy manager's level of reliability on the total secured supply in the benchmark model becomes evident in the functional form in Eq.(7). A plot of the total secured supply over the planning period in case of the other models based on the simulations carried out for the first and order Fourier approximation, is

Table 2: *Results of the regression analysis for the first and second order fit of the parameters. The optimal ex-post strategy is given for the year 2017.*

Coeff.	Model (M0)	(M1)	(M2)	(M3)	(M4)	(2017)
c_i $[GWh]$	-	6.02 (0.04)	6.08 (0.04)	6.10 (0.08)	5.89 (0.01)	6.16 (0.17)
γ_i $[GWh]$	-	-5.74 (0.06)	-5.95 (0.05)	-5.85 (0.11)	-5.55 (0.21)	-5.89 (0.22)
In sample: mean CV(RMSE) [%]	44.7	43.0	43.2	45.8	44.2	-
Out of sample: mean CV(RMSE) [%]	40.3	37.6	39.3	37.4	38.5	-

Note: standard errors in parenthesis

Significance Levels: all regression results are significant, p<0.001

illustrated in Fig.2(f) and (g). This effect is studied in a regression model $d_i = c_i + \gamma_i \chi + \varepsilon_i$, where i represents the model index, see Tab.2.

The slope coefficient γ_i can be interpreted as the "marginal costs of risk", i.e. the amount of energy in one year that is supplied less, when the energy manager's attitude towards risk increases by a small amount. The negativity of this parameter indeed shows that the total energy is decreasing when the energy manager is more risk averse. The magnitude of this effect varies according to the model choice and is the highest in model M2 which estimates approximately 5.95 GWh less in one year when the risk parameter increases incrementally.

A retrospective validation of the model is carried out by analyzing the goodness of fit in a backtest simulation, where the performance criterion is specified as the CV(RMSE). The models are evaluated according to their ability to reproduce the optimal ex-post strategy of the year 2017 and compared with the stochastic energy models. The mean CV(RMSE) for all reliability levels considered for each model is given in Tab.2. Within the universe of stochastic models, the benchmark model using Gaussian energy assets is outperformed by all models M1-M4 concerning the out of sample validation, wheres the benchmark model is outperformed by the models M1, M2 and M4 concerning the in-sample validation.

Considering the model M3, the Beta distribution works best to estimate the secured supply in a stochastic setting and reduces the CV(RMSE) of the benchmark model by approximately 8%. Gaussian energy assets can be considered as a simplification in various model aspects but can lack in an accurate description of reality, ranging from negatively supported probability density functions to symmetric and skewless descriptions of the power available of the energy assets.

This result is further investigated, by comparing the models M1-M4 with the benchmark model in terms of a one-sided two-sample test. The models are tested on the mean value of the associated CV(RMSE) compared to

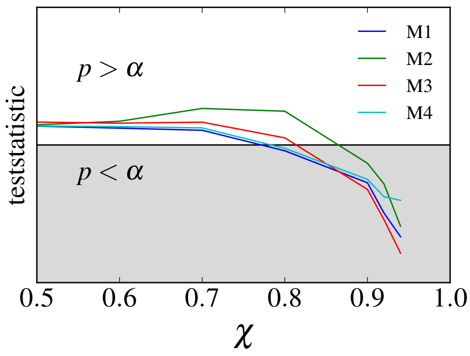

Figure 3: *The results of the test are given for all models and levels of reliability considered. Values below zero denotes significant results.*

the benchmark solution, $\mu_{\mathrm{Mi}} < \mu_{\mathrm{M0}}$, by bootstrapping $n = 10000$ samples. The test statistic

$$Z_i = \frac{\hat{\mu}_{\mathrm{Mi}} - \hat{\mu}_{\mathrm{M0}}}{\sqrt{\hat{\sigma}^2_{\mathrm{Mi}}/n + \hat{\sigma}^2_{\mathrm{M0}}/n}}$$

is approximated normally distributed under H_0, where the significance level is chosen to be $\alpha = 0.05$. The results of the test shows, that there is a difference in the performance of the models when the different levels of reliability are taken into account. While there is no evidence that the models M1-M4 perform better in the regime of low levels of reliability $\chi \lesssim 0.8$, it can be observed that all models M1-M4 perform better in the regime of higher levels of reliability $\chi \gtrsim 0.8$. This can be seen in Fig.3 as the value $Z_i + z_{1-\alpha}$ drops below zero which denotes significance, whenever the level of reliability is specifically high. These values however, are typically used in the design of reliable systems.

4 Conclusion

The aim of this paper is twofold. First of all, we introduce the framework of a stochastic planning model which incorporates the estimation of the supply that can be provided from RES, with a special focus on the security of supply, via the RBDO methodology. This can be considered as the basic building block for a modular framework which introduces stochastic supply planning of renewable energy technologies. An advantage of this formulation lies in the fact that it is formulated in terms of an optimization problem and can be incorporated in other widely used optimization problems. The basic model is illustrated by means of two applications in order to show the flexibility, which makes this framework adaptable to a great variety of stochastic planning problems. Not only variable load profiles of RES but also issues related to the unpredictability of these technologies can be considered. The supply-demand constraint is considered as a stochastic inequality which has to be true with at least a certain ex-ante chosen level of reliability. This reliability parameter reflects upon the energy manager's attitude towards risk and acts a threshold to determine the optimal strategy. Numerical solutions are obtained using methods of stochastic optimization to solve the RBDO problem based on scenario approximation and scenario reduction techniques. The proposed framework can be considered as a flexible planning tool which is able to supplement proactive managerial decisions concerning stochastic energy planning problems including RES. The application of the sample approach makes the model also accessible to general distributions.

The second goal of this paper is to compare different ways to model the power available (via Weibull, Exponential, Beta and Log-normal distributions) in the energy model. A retrospective validation based on a backtest simulation of the model's ability to reproduce the ex-post optimal strategy of the following year is performed. The results of the backtest simulation shows that the benchmark model of Gaussian distributions is outperformed by the models using a non-normal distribution to model the power available which reduce the average coefficient of variation of the root mean squared error by approximately 8%. Further investigation shows, that the performance of the model depends on the ex-ante level of reliability. The benchmark model is outperformed by all other models in the regime of risk averse energy managers with a level of reliability $\chi \gtrsim 0.8$. This gives raise to the fact, that Gaussian distributions can be considered as an approximation having the advantage of lower computational complexity, but can lack in an accurate description of reality. This reliability levels however, are typical values considered in RBDO problems, which establishes the practical use of the framework introduced.

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Appendix

Results of the Examples CCP1 and CCP2

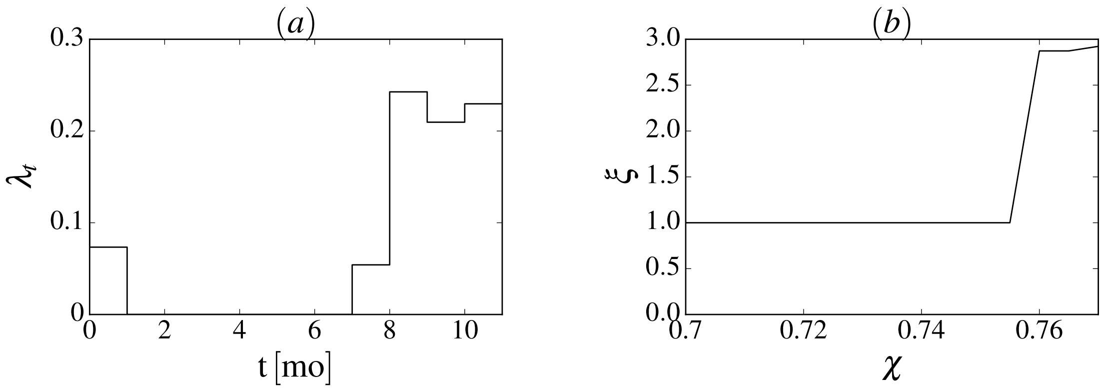

Figure 4: *Figure (a) shows the solution of the first example CCP1 and (b) shows the solution for CCP2 for varying levels of reliability.*

The solutions of the previously discussed problems CCP1 and CCP2 are given in Fig.4(a) and (b), respectively. These examples illustrate the approach, numerical values should be interpreted with care. Plot (a) shows the optimal use from a power reserve such that the RES can provide the maximum supply under the reliability constraint and an upper and lower power bound of the secured supply. Plot (b) considers the expansion problem to find the solar capacities which should be additionally installed to satisfy athe demand with certain reliability, formulated in terms of minimizing the costs. The amount to install depends on the energy manager's level of reliability specified in the model. The model is simulated for various parameters of χ to find the level where the energy manager considers to install new capacities.